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Charge symmetry breaking via ρ-ω mixing from model quark-gluon dynamics

K. L. Mitchell*, P. C. Tandy*, C. D. Roberts† and R. T. Cahill‡
* *Center for Nuclear Research, Department of Physics, Kent State University, Kent, Ohio 44242*
† *Physics Division, Argonne National Laboratory, Argonne, Illinois 60439*
‡ *School of Physical Sciences, Flinders University of South Australia, SA 5042, Australia*

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Abstract

The quark-loop contribution to the $\rho^0 - \omega$ mixing self-energy function is calculated using a phenomenologically successful QCD-based model field theory in which the ρ^0 and ω mesons are composite $\bar{q}q$ bound states. In this calculation the dressed quark propagator, obtained from a model Dyson-Schwinger equation, is confining. In contrast to previous studies, the meson-$\bar{q}q$ vertex functions are characterised by a strength and range determined by the dynamics of the model; and the calculated off-mass-shell behaviour of the mixing amplitude includes the contribution from the calculated diagonal meson self-energies. The mixing amplitude is shown to be very sensitive to the small isovector component of dynamical chiral symmetry breaking. The spacelike quark-loop mixing-amplitude generates an insignificant charge symmetry breaking nuclear force.

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1. Introduction. The on-mass-shell mixing of ρ^0 and ω mesons can be represented via a 2×2 matrix, $\mathcal{M}^2$, with diagonal elements m_ρ^2 and m_ω^2 and both off-diagonal elements equal to the mixing amplitude Π. The established value of Π, deduced from $e^+e^- \to 2\pi$ data, is -4520 ± 600 MeV^2 [1]. Recently [2] the systematics of electromagnetic and semi-strong mass splittings of the hadrons have been used to deduce a universal strength of -5000 MeV^2 for isovector mixing; i.e., for both $\rho^0 - \omega$ and $\pi - \eta$. The contributions that this mixing makes to the various charge symmetry breaking (CSB) phenomena in nuclear physics have been much studied [1,3]. The central element is the corresponding CSB contribution to the nucleon-nucleon potential, which is obtained in the boson exchange approach by using

$$\frac{-\Pi}{(k^2 - m_\rho^2)\,(k^2 - m_\omega^2)} \tag{1}$$

as the off-diagonal propagator. Without evidence of significant momentum dependence, Π has been taken as a constant in nuclear CSB applications and the resulting phenomenology has been very successful. [1,3] In a number of circumstances, the $\rho^0-\omega$ CSB nuclear potential thus obtained has been seen to dominate over other mechanisms, such as electromagnetic effects, the mass difference of neutrons and protons and $\pi^0-\eta$ mixing. A particularly striking example is the IUCF measurement [4] of the analyzing power difference in np scattering at 180 MeV.

In QCD, the mixing arises because of the small u-d current-quark-mass difference, which induces a difference between the propagation of the $\bar{u}u$ and $\bar{d}d$ components, and one expects a momentum dependent off-diagonal self-energy. Knowledge of the off-mass-shell behaviour is needed to extrapolate to spacelike momenta appropriate to meson exchange between nucleons. The simplest possible contribution to $\Pi(k^2)$ is a single quark loop and the initial investigation employed free constituent quark propagators and phenomenological meson-q-$\overline{q}$-vertex form factors to obtain a closed form expression [5]. The relevance of the result is clouded somewhat by the presence of a spurious quark production threshold, inconsistent with the notion of quark confinement. To avoid this unphysical feature, which arises near the meson mass-shell, a subsequent work [6] employed a simple, phenomenological form of dynamical quark propagator that implements confinement through the absence of a mass-shell pole. Nevertheless the central result remained the same: for spacelike momenta appropriate to vector exchange between nucleons, the mixing is so much weaker than the mass-shell value that the corresponding CSB nuclear potential is negligible. A recent QCD sum rule investigation [8] found a somewhat stronger momentum dependence, with $\Pi(k^2)$ reaching the negative of the mass-shell value at $k^2 \approx 0$. In contrast, the momentum dependence of the $\pi^0 - \eta$ mixing amplitude calculated from chiral perturbation theory appears to be considerably weaker [7].

There are a number of issues that suggest the quark loop calculations may not yet be reliable for this process. An important consideration for $\rho^0 - \omega$ mixing is that it is driven by the small isovector component of the quark propagator, which in turn is driven by the isovector current mass $m_u - m_d$. A direct parametrisation of a propagator model is usually constrained by a fit to quantities related to dynamical chiral symmetry breaking (DCSB), such as f_π, $< \bar{q}q >$ and m_π. These are isoscalar constraints and are not sufficient here. The explicit chiral symmetry breaking (ECSB) generated by current masses is weaker but its role is magnified in the isovector propagator and some dynamical guidance is to be preferred.

The low momentum behaviour of the dressed quark propagator can be calculated using a dynamical model Dyson-Schwinger equation (DSE). However, typical DSE studies are carried out numerically in Euclidean space; i.e., at spacelike momenta, where maximal use can be made of the constraints from perturbation theory and the renormalisation group in QCD [9,10]. Such numerical studies do not provide readily accessible information about the behaviour at real, time-like quark momenta nor the domain of complex momenta sampled in a Euclidean treatment of meson propagators at time-like meson momenta.

The above issues are addressed in this work through use of a recently developed solution [11] of a model DSE that yields the quark propagator in closed form as an entire function in the complex momentum plane. This structure can be interpreted as representing confined quarks [9,12] and provides an unambiguous representation over the complex momentum domain required by the quark loop. The dependence upon the current mass is explicit in the model and is produced by the DSE dynamics.

We calculate the mixed $\rho^0-\omega$ self-energy function produced by the vector meson sector of a QCD-based model field theory [13,14], the Global Colour-symmetry Model (GCM) [for a review see [15]], which formalises the coupled Dyson-Schwinger–Bethe-Salpeter equation approach to QCD phenomenology [9] and allows several internally consistent features to be implemented for the first time. The off-mass-shell behaviour we obtain for the mixing amplitude includes the contribution from the off-mass-shell structure of the composite ρ and ω propagators. In addition, the strength and range parameters used to model the meson-$\bar{q}q$ Bethe-Salpeter amplitudes are determined by the meson mass-shell dynamics within the GCM. This eliminates the need for several assumptions made in previous studies. A short account of some of this work has recently been presented [16].

2. $\boldsymbol{\rho^0}$-$\boldsymbol{\omega}$ Mixing Amplitude. The description of the ρ^0 and ω mesons as $\bar{q}$-q composites is obtained from a phenomenologically successful, QCD-based model field theory [13,14], which is defined by the action:

$$S[\bar{q},q] = \int d^4x\, \bar{q}(x) \left(\gamma\cdot\partial_x + M\right) q(x) + \tfrac{1}{2}\int d^4x d^4y\, j^a_\mu(x) g^2 D(x-y) j^a_\mu(y), \tag{2}$$

where M is the current-quark-mass matrix, $j^a_\mu(x) = \bar{q}(x)\frac{\lambda^a}{2}\gamma_\mu q(x)$ is the quark current, and $D(x-y)$ is a model effective-gluon-propagator in a Feynman-like gauge. Here we consider only u and d flavors and use a Euclidean metric throughout such that $a\cdot b = a_\mu b_\mu$ and $\{\gamma_\mu,\gamma_\nu\} = 2\delta_{\mu\nu}$, with $\gamma_\mu = \gamma_\mu^\dagger$.

After bosonisation and expansion about the classical vacuum [by which is meant the minimum of the action], the tree-level effective action for the mesons is [17]

$$\begin{aligned}\hat{S} = Tr\sum_{n=2}^{\infty} \quad & \frac{(-1)^n}{n}[G_0(i\gamma_5\vec{\tau}\cdot\vec{\pi} + i\gamma_\nu\omega_\nu + i\gamma_\nu\vec{\tau}\cdot\vec{\rho}_\nu + \cdots)]^n \\ & + 9\int d^4x d^4y \frac{\frac{1}{2}\vec{\pi}\cdot\vec{\pi} + \omega^2 + \vec{\rho}\cdot\vec{\rho} + \cdots}{2g^2 D(x-y)},\end{aligned} \tag{3}$$

where the meson fields are bilocal fields corresponding to bilocal combinations of quark fields; for example, $\omega_\mu(x,y) \sim \bar{q}(y) i\gamma_\mu q(x)$. The dressed quark propagator, G_0, that appears here has a self-energy determined by the classical vacuum configurations of the bilocal fields. We use a momentum representation in which q is conjugate to $x-y$ and

P is conjugate to $(x+y)/2$. The vector bilocal fields are represented by, for example, $\gamma_\mu \omega_\mu(q;P) = \Gamma(q;P)\gamma_\mu T_{\mu\nu}(P)\omega_\nu(P)$, where $T_{\mu\nu}(P) = \delta_{\mu\nu} - P_\mu P_\nu/P^2$ and $\Gamma(q;P)$, a Lorentz-scalar function, is the dominant Bethe-Salpeter amplitude in the vector $\bar{q}q$ channel [17,18].

We are interested in tree-level coupling in the $\rho^0 - \omega$ sector. In a matrix notation where V_μ denotes the column of transverse fields $(\vec{\rho}_\mu, \omega)$, the GCM action up to second order in the fields is

$$\hat{S}[\rho,\omega] = \frac{1}{2}\int \frac{d^4P}{(2\pi)^4} V_\mu^T(-P)\Big[\Delta_{\mu\nu}^{-1}(P) + \Pi_{\mu\nu}(P)\Big]V_\nu(P), \tag{4}$$

where Δ^{-1} is diagonal and Π is the off-diagonal $\rho^0 - \omega$ self-energy. The diagonal inverse propagator is

$$\Delta_{\mu\nu}^{-1}(P) = \int \frac{d^4q}{(2\pi)^4} tr\left[G_0(q_-)i\gamma_\mu f G_0(q_+)i\gamma_\nu f\right]\Gamma^2(q;P) + 9\delta_{\mu\nu}\int d^4r \frac{\Gamma^2(r;P)}{g^2D(r)}, \tag{5}$$

with tr denoting a trace over spin, flavor and color. Here f are the relevant flavor factors [1 for the ω, and $\vec{\tau}$ for the $\vec{\rho}$] and $q_\pm = q \pm P/2$, where P is the meson momentum and q is the loop momentum associated with the $\bar{q}q$ substructure. The mixed self-energy, $\Pi_{\mu\nu}(P)$, is given by an expression identical to the first term of (5) except that one of the flavor factors is τ_3 and the other is 1, thus it is nonzero only if G_0 has an isovector component.

At tree level the diagonal terms are the same for ρ^0 and ω and the general form of the transverse component is

$$\Delta_{\mu\nu}^{-1}(P^2) = T_{\mu\nu}(P)\,(P^2 + M_V^2)\,Z(P^2), \tag{6}$$

where the position of the zero defines the physical mass M_V. The meson fields may be rescaled so that at least the on-mass-shell value of $\sqrt{Z}$ is absorbed into the fields to produce a physical normalization. We elect to absorb $\sqrt{Z(P^2)}$ into the fields so that the diagonal propagators have the standard form characterised only by the physical mass. The corresponding mixed self-energy $\Pi_{\mu\nu}(P)/Z(P^2)$ then contains the off-mass-shell structure appropriate for use in meson exchange applications that use the standard point-meson form for diagonal propagators. The net result of this is that the renormalised version of (5) is obtained via the replacement:

$$\Gamma(q;P) \to F(q;P) = \frac{\Gamma(q;P)}{\sqrt{Z(P^2)}}. \tag{7}$$

The resulting vertex function, $F(q;P)$, contains the strength of the meson-$\bar{q}q$ coupling as determined by the model. On the mass-shell this is equivalent to imposing the standard physical normalisation for a Bethe-Salpeter amplitude [19].

The corresponding mixed self-energy

$$\Pi_{\mu\nu}(P) = \int \frac{d^4q}{(2\pi)^4} tr\left[G_0(q_-)i\gamma_\mu \tau_3 G_0(q_+)i\gamma_\nu\right]F^2(q;P) \tag{8}$$

has a transverse component, $\Pi_{\mu\nu}(P) = T_{\mu\nu}(P)\Pi_T(P^2)$, which yields the mixing amplitude as the following difference of quark loop integrals:

$$\Pi_T(P^2) = \Pi_T^u(P^2) - \Pi_T^d(P^2). \tag{9}$$

With the quark propagator in the form $G_0(p) = -i\gamma\cdot p\sigma_V(p^2) + \sigma_s(p^2)$, the required quark loop integrals are

$$\Pi_T^i(P^2) = \tag{10}$$
$$-\frac{12}{(2\pi)^3}\int_0^\infty ds s F^2(s;P^2)\int_{-1}^{+1} du(1-u^2)^{\frac{1}{2}}\Big[\big(\tfrac{s}{3}(1+2u^2) - \tfrac{1}{4}P^2\big)\sigma_v^i(+)\sigma_v^i(-) + \sigma_s^i(+)\sigma_s^i(-)\Big],$$

where $s = q^2$, $u = \hat{q}\cdot\hat{P}$ and $\sigma(\pm)$ denotes $\sigma(q_\pm^2)$, where $q_\pm^2 = s + P^2/4 \pm u\sqrt{sP^2}$. For time-like meson momentum, where $P^2 < 0$, $\sqrt{P^2} \to i\sqrt{-P^2}$ and $\sigma(-) = \sigma^\star(+)$. By assumption the vertex function, F, is independent of the angle u.

3. Quark propagator. Because the momentum region of interest $[P^2 \geq -m_\omega^2]$ includes a time-like sector, the loop integral (10) requires knowledge of the propagator amplitudes $\sigma_s(z)$ and $\sigma_v(z)$ in the domain of the complex $z = x + iy$ plane enclosed by the parabola $|y| = m_\omega\sqrt{x + m_\omega^2/4}$ with $x \geq -m_\omega^2/4$. A fit to soft chiral quantities [such as $\langle\bar{q}q\rangle$, f_π, m_π, r_π] does not probe a propagator model over this domain. [The corresponding quark loop appropriate for pion physics also requires knowledge of the quark propagator in the complex plane but in a region whose area is reduced by a factor of $(m_\omega/m_\pi)^2 \approx 25$ and hence the chiral limit ($m_\pi \to 0$) captures the dominant physics.]

The quark propagator given by Ref. [11] provides a behaviour in the complex domain that is accountable to DSE dynamics. It is conveniently expressed in terms of dimensionless amplitudes $\bar{\sigma}_s(x) = \lambda\sigma_s(q^2)$ and $\bar{\sigma}_v(x) = \lambda^2\sigma_v(q^2)$, where $x = q^2/\lambda^2$ and λ is the momentum scale. The amplitudes $\bar{\sigma}$ are also explicit functions of the dimensionless current-quark mass $\bar{m} = m/\lambda$. This DSE model employs a momentum-space delta function for the gluon propagator, with a strength set by λ^2, and a dressed quark-gluon vertex [20], which is compatible with the Ward-Takahashi identity. The amplitudes $\bar{\sigma}$ are entire functions in the complex momentum plane with an essential singularity at timelike infinity. The exact expressions are a little complicated when the current mass is included. We have verified that for typical current masses [~ 10 MeV], the following simple forms

$$\bar{\sigma}_s(x) = c(\bar{m})e^{-2x} + \frac{\bar{m}}{x}(1 - e^{-2x}), \tag{11}$$

$$\bar{\sigma}_v(x) = \frac{e^{-2x} - (1-2x)}{2x^2} - \bar{m}\; c(\bar{m})e^{-2x}, \tag{12}$$

are accurate representations [to order 10^{-4}] on the domain of complex momenta encountered in the quark loop.

These forms also accurately represent the leading behaviour of the quark propagator at large spacelike q^2 in the model and in QCD. The constant $c(\bar{m})$, which represents the strength of DCSB, is not determined by the model DSE solution; the reason being that the model of Ref. [11] has only two scales [momentum λ and mass m]. A third scale is necessary to fix $c(\bar{m})$. In a more elaborate model, such as that of Ref. [10], this is provided by Λ_{QCD}, which is introduced by the inclusion of the known large spacelike-q^2 behaviour of the gluon propagator. However, a numerical solution is then necessary and the extension

into the complex momentum plane is not unique. We consider (11) and (12), with $c(\bar{m})$ a parameter to be determined below, to provide a realistic model propagator for a confined quark in QCD over the region of the complex plane sampled in (10).

To fix the values of the parameters λ and $c_0 = [c_u + c_d]/2$ we follow Ref. [21] and use (11) and (12) to fit $\langle \overline{q}q \rangle$, f_π, r_π and the π-π scattering lengths: a_0^0, a_0^2, a_1^1, a_2^0. With $(m_u + m_d)/2 = 7.5$ MeV this leads to best fit values of $\lambda = 0.893$ GeV and $c_0 = 0.523$, which yield $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = (183\ \mathrm{MeV})^3$, $f_\pi = 90$ MeV, $r_\pi = 0.56$ fm and a mean deviation of 19% from the experimental $\pi\pi$ scattering lengths.

4. Bethe-Salpeter amplitude. The dominant vector meson Bethe-Salpeter amplitude, $F(q;P)$, also appears in (10). This may be calculated in the GCM using a variational form of the off-mass-shell generalised-ladder-approximation Bethe-Salpeter eigenvalue equation [15], and a simple, reasonable approximation for the shape on the mass-shell is provided by the P-independent form $\Gamma(q;P) = \exp(-q^2/a^2)$, with $a = 0.62$ GeV. To simplify the computations here the renormalisation function, $Z(P^2)$, was obtained from (5) after the second [constant] term is set to produce the vector mass 783 MeV. The normalised amplitude $F(q;P)$ is then obtained from (7).

5. Isovector component of the quark propagator and $\boldsymbol{\rho^0}$-$\boldsymbol{\omega}$ mixing. It only remains now to set the isovector component of the quark propagator, which is a measure of the isovector character of DCSB. With the representation $G_0^{-1}(q^2) = i\gamma \cdot qA(m,q^2) + B(m,q^2)$, one commonly used assumption is that $A(m,q^2) \approx A(0,q^2)$ and $B(m,q^2) \approx B(0,q^2) + m$. This is the procedure followed in the one other calculation of $\rho^0 - \omega$ mixing that used a dynamical quark propagator [6]. It is convenient because only the chiral limit amplitudes need to be modelled. With the $\overline{m} = 0$ forms of A and B extracted from (11) and (12), this yields the prediction for the $\rho^0 - \omega$ mixing amplitude shown as the dotted line in Fig. 1. [Throughout this work we use $m_d - m_u = 5$ MeV.]

The momentum dependence obtained is quite similar to that found in earlier work [5,6]. The experimental mass-shell mixing strength was produced in the work of Ref. [6] with use of phenomenological meson-quark coupling strengths $g_\rho = 4$ and $g_\omega = 5$ deduced by simple scaling arguments from empirical meson-nucleon coupling constants. In the present approach, however, the vertex strength is an outcome of the model, and the comparable measure from (7) is $g_\rho = g_\omega = F(q = 0; P^2 = -M_V^2)$ which produces the larger value 16.4. Nevertheless, we obtain only 50% of the experimental mixing value. The quark vertex alone is not an accurate guide to the net coupling strength between hadrons because it does not account for the particular way the hadron dynamics samples the quark propagator amplitudes, especially the wave function renormalisation function. We feel the mass-shell condition used here provides the internally consistent vertex normalisation for the meson model.

The above estimate for the isovector propagator can be improved through the use of solutions from a realistic model DSE to directly determine the isovector component of $c(\overline{m})$ in (11) and (12). [We note that in realistic DSE studies $A(m,0)$ decreases with increasing m and $B(m,0)$ is badly underestimated by $B(0,0)+m$.] From the model of Ref. [10], which uses a model gluon propagator that is the sum of a momentum-space delta function and the one-loop renormalisation group result together with a dressed quark-gluon vertex, one infers a value of $c_1 = (c_u - c_d)/2 = 0.006$ for $m_u = 5$ MeV and $m_d = 10$ MeV [by matching

$c(\overline{m})$ to the $\overline{m}$ dependence of the solutions at $q^2 = 0$ and using the $C = 500$, $\tau = e$, $N_f = 4$ results from Ref. [10]]. This is an indication of the size of the isovector component of DCSB, which is seen to be a 1% effect in this case.

The mixing amplitude obtained with this parameter set is given by the dashed curve in Fig. 1: less than 25% of the mass-shell mixing strength is accounted for. Of course, the above arguments are simply a guide. If c_1 is treated as a free parameter a fit to the mass-shell mixing amplitude requires $c_1 = 0.009$, which yields the momentum dependence shown by the solid line in Fig. 1. Clearly, the quark loop mixing mechanism is extremely sensitive to the small isovector component of DCSB.

Qualitatively, the momentum dependence of the mixing amplitude is insensitive to the form of the model Bethe-Salpeter amplitudes at the vertices. The insert in Fig. 1 illustrates this by comparing the previously mentioned result [solid line] from the simple gaussian amplitude to the result [dash-dot line] obtained using the dominant ρ-amplitude obtained in a recent Bethe-Salpeter calculation [18]. Both representations for the on-mass-shell meson vertex give essentially identical results.

In Fig. 2 we isolate the part of the off-mass-shell behaviour of the mixing amplitude that is produced by the momentum dependence of the function $Z(P^2)$. This is a measure of the extent to which the diagonal meson propagators depart from the standard point form due to their $\bar{q}q$ substructure. The solid curve is the result previously shown and includes the off-shell contributions from the propagators, while the dashed curve has $Z(P^2)$ held fixed at the mass-shell value to remove them. Clearly this contribution is significant for $P^2 > 1$ GeV^2.

6. CSB potential. In Fig. 3 we display the CSB NN potential $V_{\rho\omega}(r)$ calculated from the standard momentum space form [1]

$$V_{\rho\omega}(P) = -\frac{g_{\rho N}\, g_{\omega N}\, F_{\rho N}(P^2)\, \Pi_T(P^2)\, F_{\omega N}(P^2)}{(P^2 + m_\rho^2)(P^2 + m_\omega^2)} \ . \tag{13}$$

In accordance with the Bonn meson exchange NN interaction model [22], monopole form-factors for ρNN and ωNN coupling are used with a common range of 1.5 GeV, and the coupling constants are taken to be $g_{\rho N}^2/4\pi = 0.41$ and $g_{\omega N}^2/4\pi = 10.6$. The mixing amplitudes in Fig. 2 give rise to the potentials shown by the solid and dotted curves in Fig. 3. The small difference between them indicates that the off-shell structure of the diagonal propagators of the composite vector mesons does not make a signficant contribution to the potential. This is because $Z(P^2)$ departs significantly from its mass-shell value only for momenta that are well suppressed by the meson-nucleon form factors. Variations of $\pm 20\%$ in the form-factor range do not change this conclusion. The dashed curve is the result that follows from the mass shell assumption, $\Pi_T(P^2) \approx \Pi_T(-M_V^2)$.

The typical quark loop result for the nuclear CSB potential is very much weaker than the potential produced by the mass-shell assumption because the mixing amplitude near $P^2 \approx 0$ is about four times smaller than the empirical mass-shell value. This is a general consequence of a node at very low momentum and an almost linear connection with the mass-shell point.

There are uncertainties in QCD-models at this scale but our investigations have shown no indications that the main conclusion above is affected. For example, the off-mass-shell behaviour of the meson-quark vertex is not guaranteed to be represented well by a single

mass-shell Bethe-Salpeter amplitude. In this regard, we have employed an estimate for the relevant $P^2 > 0$ extrapolation of the gaussian representation of $\Gamma(q;P)$ described earlier. This can be defined within the GCM by the variational approach [15] to the eigenvalue problem based on (5). The dominant effect is incorporated by a momentum-dependent range $a(P^2)$ for the q^2 behaviour that increases with increasing spacelike momentum. The net result is an increased spacelike suppression of the mixing amplitude with a typically smaller value at $P^2 \approx 0$.

7. Quark loop generates weak mixing. The quark loop mechanism will, in general, produce a weak mixing at small spacelike momenta. This can be seen from an analysis of the loop integral in (9) and (10) at first order in the isovector components of σ_v and σ_s for $P^2 > 0$.

From (12), both the isoscalar and isovector σ_v are always positive. Thus the vector contribution to $\Pi_T(P^2)$ is positive for sufficiently large $P^2 > 0$. From (11), the isoscalar σ_s is always positive, while the isovector component has a small positive contribution from the first term, and a dominant negative contribution from the second term. Thus the scalar contribution to $\Pi_T(P^2)$ is also positive. Since the vector contribution is dominant, there will be a change in sign at small momenta driven mostly by the sign change in the coefficient of the vector part of the integrand in (10). The position of the node is approximately $P_0^2 \approx < 4s(1+2u^2)/3 >$ with the expectation value estimated from the dominant behaviour of the remaining integrand in (10). Using only the typical momentum scales that characterise the propagator and the vertex, we obtain $P_0^2 \approx 0.5$ GeV^2 which is in accord with the numerical results.

On the time-like side of the node, the linear factor of P^2 in (10) represents the behaviour well.

These observations hold equally well if a free constituent quark propagator is used.

8. Summary and conclusions. We have calculated the quark loop contribution to the ρ^0-ω mixing amplitude in a way that retains, for the first time, a number of self-consistent features that follow from use of a QCD-based model field theory. These include DSE model guidance for the quark propagator at the time-like and complex momenta that arise, calculation of the strength and range of the vertex function within the model, and the contribution to the off-shell behaviour arising from the composite nature of the diagonal meson propagators.

The mass-shell value of the mixing amplitude is found to be very sensitive to the isovector component of DCSB. For the behaviour at spacelike momenta relevant to the CSB component of the NN interaction, none of the new elements we include alter the conclusion that the quark loop mechanism alone generates an insignificant CSB potential.

This failure suggests that other isospin-symmetry breaking intermediate states, such as the virtual pion-loop process: $\rho \to \pi\pi \to \omega$ through a G-parity violating $\omega\pi\pi$ vertex, may contribute significantly to $\Pi_T(P^2)$. In this connection, and in the context of the field theoretical model considered herein, the $\omega\pi\pi$ vertex occurs as a quark loop which is non-zero for $m_u \neq m_d$ and a recent estimate [16] indicates that $g_{\omega\pi\pi}/g_{\rho\pi\pi} \approx \Pi_T(-M_V^2)/m_\omega^2$. It is therefore possible that the pion loop mechanism could make a significant contribution to $\rho^0 - \omega$ mixing and the associated NN CSB potential should be investigated. This is especially true since the imaginary part of such an amplitude relates to the same interference effect in

the 2π decay channel that is the basis of the accepted experimental value of $\rho^0 - \omega$ mixing.

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FIGURES

FIG. 1. The quark loop contribution to the $\rho^0 - \omega$ mixing amplitude. Here $P^2 > 0$ represents spacelike momentum. The solid curve is obtained with the quark propagator parameter c_1 adjusted to fit the experimental mixing value; the dashed curve uses the value inferred from a realistic DSE solution; the dotted curve is produced by the common assumption that a current mass provides a constant shift to the scalar self-energy function. The insert demonstrates that the mass-shell vector Bethe-Salpeter amplitude does not introduce significant model dependence.

FIG. 2. The $\rho^0 - \omega$ mixing amplitude with (solid curve) and without (dashed curve) the off-mass-shell contribution from the calculated diagonal propagators of the $\bar{q}q$ mesons.

FIG. 3. The $\rho^0 - \omega$ mixing contribution to the CSB NN potential form factor. The dashed curve is obtained under the assumption that the mixing amplitude is a constant given by the mass-shell value. The quark loop calculation is shown by the solid curve when the off-mass-shell contribution from the diagonal meson propagators is included, and by the dotted curve when it is not.